Why the √3?

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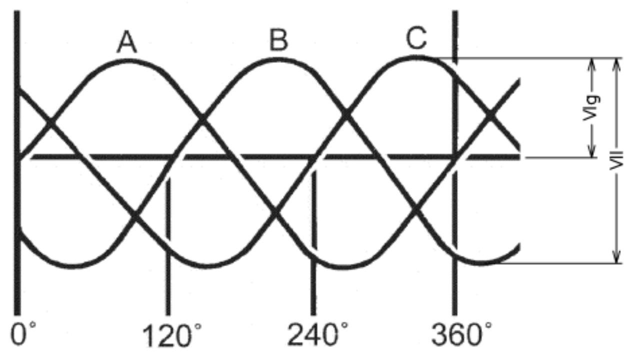
Introduction

In electrical engineering calculations involving three phase power systems, there is a mysterious √3 that always seems to work its way into the calculations. Where did it come from, what does it represent, and why do we use it?

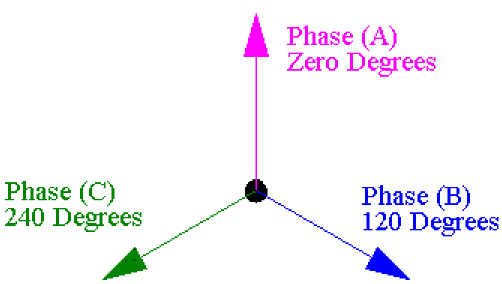
Background



Before we delve into answering these questions, it is important to touch on a little bit of electrical theory regarding three phase systems. In a 3-phase system, there are three sinusoidal waves, each separated by 120 degrees as shown to the right. This sequence is repeated 60 times a second in a 60 Hz power system. The horizontal axis is considered ground or zero. The magnitude of each of the three phases above or below the horizontal axis is called the *line to ground voltage*, which we represent as V_{LG}. The magnitude of a positive peak of one phase above the above the horizontal axis and the negative peak of a different phase below the horizontal axis is called the *line to line voltage*, which we represent as V_{LL}.

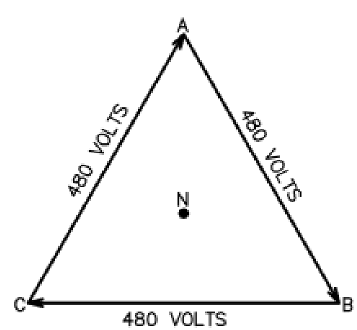
It is important to understand that in a 3 phase system, the neutral point is ground.

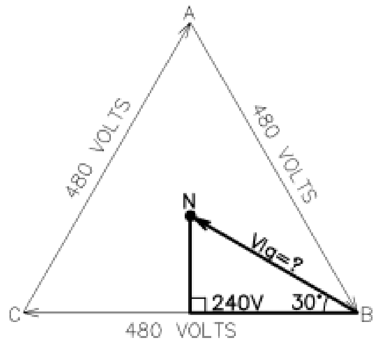
Where Did It Come From?



Let's look at a phasor diagram of the sinusoidal waveforms of a typical three phase power system, as shown in the figure to the left. The dot in the middle represents the neutral point (ground).

Next, we take that phasor diagram and set it up to add the phase vectors by putting the tail end of Phase B at the arrow end of Phase A and putting the tail end of Phase C at the arrow end of Phase B, the results are shown to the right. You could also look at the phasor diagram above and draw lines between the arrowheads. The results are an equilateral triangle with the neutral point still in the middle. Let's assign a magnitude of 480 volts to the phase vectors for demonstration purposes. This magnitude is the line to line voltage.





If we drop a line from the neutral point down perpendicular to phase vector BC, it intersects the vector at the midpoint, which is 240 volts. And if we draw a vector between points B and N, this will represent the line to ground voltage. This results in angle CBN measuring 30 degrees.

You could choose any phase vector to work this with and would come up with the same result.

Using a little bit of trigonometry we can figure out the magnitude of vector BN, which is the line to ground voltage or V_{LG}.

$\cos(30) = 240 \text{ volts} \div V_{LG}$
$V_{LG} = 240 \text{ volts} \div \cos(30)$
$= 277 \text{ volts}$

Does the answer look familiar? In a 480 volt, three phase, 4 wire power system, the line to ground voltage is 277 volts. If you work this using a 208 volt system, a 600 volt system, or a 400 volt system, you will come up with the line to ground voltages we all know. As a matter of fact, you can use this knowledge to find the line to ground voltage of any three phase system, as long as you know the line to line voltage, by just plugging the line to line voltage into the following formula:

$V_{LG} = (V_{LL} \div 2) \div \cos(30)$ which can be simplified to
$V_{LG} = V_{LL} \div 2\cos(30)$

Looking at only the term 2cos(30) calculated out, you get 1.732... Square that number and what do you get? Three! Therefore,

$V_{LG} = V_{LL} \div \sqrt{3}$

Conversely, if you know the line to ground voltage, you can rearrange the formula to calculate the line to line voltage.

$V_{LL} = V_{LG} \times \sqrt{3}$

Using the example above:

$277 \text{ volts} \times \sqrt{3} = 480 \text{ volts}$

What Does It Represent?

After going through all of the work above, what the √3 represents should be readily apparent. It is simply the relationship or the ratio of the line to line voltage to the line to ground voltage.

$V_{LL} \div V_{LG} = \sqrt{3}$

Why Do We Use It?

Put simply, we use it to convert between phase voltage and line voltage or vice versa.

$V_{LL} \div \sqrt{3} = V_{LG}$ or;
$V_{LG} \times \sqrt{3} = V_{LL}$

The sum of the line to ground voltages in a power system results in the total voltage available in the system, which we call the *effective voltage*. For example, in a 120/208 volt, 3 phase power system, the effective voltage is 120 volts × 3 = 360 volts. Therefore, the effective voltage for any three phase power system can be summarized as:

$$V_{EFF} = 3 \times V_{LG}$$

Looking again at the formula for determining the line to ground voltage from a known line to line voltage we have:

$$V_{LG} = V_{LL} \div \sqrt{3} \qquad \textbf{FORMULA 1}$$

And if we look again at the formula for determining the effective voltage of a 3 phase system and rearrange it a bit, we have:

$$V_{LG} = V_{EFF} \div 3 \qquad \textbf{FORMULA 2}$$

Now if we substitute the **FORMULA 2** into **FORMULA 1**, we have:

$$V_{EFF} \div 3 = V_{LL} \div \sqrt{3}$$

Now to solve for V_{EFF}:

$$\begin{aligned} V_{EFF} \div 3 &= V_{LL} \div \sqrt{3} \\ V_{EFF} &= 3 \times (V_{LL} \div \sqrt{3}) \\ &= 3V_{LL} \div \sqrt{3} \\ &= 1.732\ldots \times V_{LL} \end{aligned}$$

There's that 1.732 number again, which we have already established is equivalent to √3. Therefore, the effective voltage of a three phase power system, which is the total voltage available in a three phase power system, can be summarized as:

$$V_{EFF} = V_{LL} \times \sqrt{3}$$

Simple, right? Now you should know where the √3 comes from and why we use it; to convert from phase voltage to line voltage, to convert from line voltage to phase voltage, and to calculate the effective voltage. Now to make things easy, a table that includes commonly encountered 3 phase voltage systems and their associated line to line voltages, line to ground voltages, and effective voltages follows.

Voltage System	V_{LL}	V_{LG}	V_{EFF}
120/208 volt, 3 phase, 4 wire wye	208 volts	120 volts	360 volts
277/480 volt, 3 phase, 4 wire wye	480 volts	277 volts	831 volts
347/600 volt, 3 phase, 4 wire wye	600 volts	347 volts	1039 volts
230/400 volt, 3 phase, 4 wire wye	400 volts	230 volts	693 volts

About Jason

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About Greg

Gregory Schnackel, P.E., LEED AP has been involved in the design of mechanical, electrical, plumbing, fire protections and information technology systems for malls, mixed-use developments, corporate offices, national retail rollouts, schools, hospitals, medical facilities, commercial and institutional buildings for over 40 years with Schnackel Engineers. Email Greg at gschnackel@schnackel.com.